Math 315 Sections 1 and 2 | Final | Name ________________
20 and 24 April 2007 | Show relevant work!
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1. Show that if $\lim a_n = a$ and $\lim b_n = b$, then $\lim a_n b_n = ab$.

2. Show that the Nested Interval Property implies the Axiom of Completeness.

3. Show that if $0 < r < 1$, then $\lim r^n = 0$.

4. Define what it means for a sequence to be Cauchy and show that a Cauchy sequence converges.

5. Prove that a closed interval has the property that if it covered by a collection of open sets, then some finite sub-collection of the open sets covers.

6. If f and g are differentiable functions with domain all real numbers. If $g(x) \neq 0$, prove that $F(x) = \frac{f(x)}{g(x)}$ is differentiable at c and find the derivative

7. Prove the uniform limit of continuous functions is continuous.

8. Show that if *f* is a differentiable, real-valued function defined for all real numbers with a bounded derivative, then *f* is uniformly continuous.

9. Prove that if a power series $\sum_{n=0}^{\infty} a_n x^n$ converges at some point x_0, then it converges absolutely for any *x* satisfying $|x|<|x_0|$.

10. Prove that if a power series $\sum_{n=0}^{\infty} a_n x^n$ converges for all $x \in (-R,R)$, then the differentiated series $\sum_{n=1}^{\infty} n a_n x^{n-1}$ converges as well for all $x \in (-R,R)$.

11. If $P_n(x)$ is the n$^{\text{th}}$ degree Taylor polynomial for *f*(*x*), and $f(x) = P_n(x) + E_n(x)$, find $E_n(0), E_n^{(1)}(0), E_n^{(2)}(0), E_n^{(3)}(0), \ldots, E_n^{(n)}(0)$ and a formula for $E_n^{(n+1)}(x)$. Explain your reasoning. ($E_n^{(k)}(x)$ is the k^{th} derivative of $E_n(x)$.)

12. Define the lower and upper integrals of a function.

13. Prove that if a function f is continuous on a closed interval $[a, \mathrm{b}]$, then it is integrable.

14. State and prove one part (you choose) of the Fundamental Theorem of Calculus.

15. Assume that the functions f and $|f|$ are integrable on the interval $[a, b]$. Show $\left|\int_a^b f\right| \le \int_a^b |f|$.

16. Let f be a differentiable function defined on $[a, b]$ so that the derivative f' of f is continuous. If $P = \{x_0, x_1, x_2, \ldots, x_n\}$ is a partition of $[a, b]$,

show $\sum_{k=1}^{n} |f(x_k) - f(x_{k-1})| \le \int_a^b |f'|$.